# PMU-Based Technique for the Estimation of Line Parameters in Three-Phase Electric Distribution Grids

**Conference Paper** · September 2018

DOI: 10.1109/AMPS.2018.8494886



Carlo Muscas
Università degli studi di Cagliari


Paolo Attilio Pegoraro
Università degli studi di Cagliari


Alexandra von Meier
University of California, Berkeley

# Handling Instrument Transformers and PMU Errors for the Estimation of Line Parameters in Distribution Grids

Paolo Attilio Pegoraro, Paolo Castello, Carlo Muscas
*Departement of Electrical and Electronic Engineering*
University of Cagliari
Cagliari, Italy
paolo.pegoraro@unica.it

Kyle Brady, Alexandra von Meier
*Department of Electrical Engineering and Computer Science*
University of California, Berkeley
Berkeley, California, USA
kwbrady@berkeley.edu

***Abstract*— The knowledge of line parameters is fundamental for building a network model that can become the basis for all the monitoring and management applications. Network line impedances are employed, for instance, for the state estimation or for protection applications in relay setting. The Phasor Measurement Units (PMUs), thanks to the synchronized voltage and current phasor measurements, provide the accurate information that can be used to evaluate the network parameters of the monitored branches. Nevertheless, the impact of instrument transformers (ITs) makes the parameter estimation a hard task. For this reason, in this paper an estimation paradigm is proposed that includes the estimation of both systematic errors (mainly due to ITs) and line parameters. The proposed method, particularly focused on distribution systems, is designed to perform simultaneous estimations across different branches of the network, aiming at exploiting the links between the involved parameters, due to topology and operating conditions. The estimation is based on a sound and detailed model of the uncertainty sources.**

***Keywords*— *power system measurements, voltage and current transducers, electric network line parameters, phasor measurement unit PMU, weighted least squares***

## I. INTRODUCTION

Phasor Measurement Units (PMUs) are one the most important elements in modern power networks monitoring and are expected to play an important role in the transformation of the electric grids from the traditional management strategies towards automation. PMUs measure, in a synchronized way, voltage and current phasors at different points in the network and are the basis to build distributed and coordinated measurement systems.

Measurements by PMUs placed on the field can be used for different applications: from local monitoring to state estimation, from voltage stability assessment to load shedding [1]. One of the emerging fields is network model estimation. Line parameters are crucial to build a good model of the power system. Network line impedances are important to allow accurate data estimation from the monitoring applications (state estimation) or to determine the correct values to be used in the protection applications (relay protection). Traditional approaches are based on nominal values, theoretical calculations and off-line measurements. In the analytical calculation, the physical and the electrical parameters are used within empirical formulas, while, in the measurement campaign, the information is usually obtained directly from the line in off-line situations [2], [3]. These methods are characterized by a lack of accuracy. This weakness in the estimation of the network parameters may have different reasons. Inaccurate manufacturing data, network changes not often updated or weather conditions can influence the estimation [3]. The values of network parameters used by the utilities can differ from the actual values from 25 % to 30 % [4].

Nowadays, to obtain an updated estimation of the network parameters, there is growing interest in the possibility of an on-line estimation. The electrical quantities used to evaluate the network parameters can be provided, for instance, by PMUs. The PMU, thanks to the synchronization and to the high accuracy and reporting rate, is already deemed to improve distribution system state estimation [5]. Such strengths make it also an interesting candidate tool to perform network parameters estimation.

PMU measurements are acquired from different points of the electric grid and are collected and aligned by the Phasor Data Concentrators to forward them to the control center [1] where processing routines run. This seems an ideal framework to exploit the coordinated data also for network model estimation. Nevertheless, measurements from PMUs can be contaminated with spikes and complex noise behavior [6], which should be mitigated before usage.

Above all, the main problem is that PMU measurements, notwithstanding the high accuracy of the device, are obviously affected by transducer (mainly instrument transformers, IT) errors that can have a dominant effect. All these issues make network parameters estimation using PMU data a challenging task.

In [7], a preliminary investigation is performed to determine the feasibility of the calculation of the impedance of both a transformer and a line from the measurements provided by PMUs for distribution networks. The method is suitable to recognize areas where the network model is inaccurate [7], but the uncertainty of the measurement devices and transducers is not included in the model.

In this paper, to improve and generalize this kind of approach, keeping into account the role of the systematic and random errors in the estimation, a new methodology is proposed. The technique relies on a PMU monitoring system and on a weighted least square approach that keeps into account the different uncertainty sources and allows the estimation of the line parameters together with the systematic deviations (mainly from ITs). The estimation method is designed to operate on multiple branches simultaneously and in a coordinated manner, thus exploiting the constraints given by network topology, load conditions, and zero-injections in order to improve the estimation.

## II. Network Parameter estimation

### A. Network and Measurement Modelling

In the following, the analysis is focused on a single line modeling. Even if a distribution line can be in general three/two- or even single-phase, the underlying concepts are the same. Thus, the discussion is here kept simple, in order to show the key points and issues. In this section, the single branch case is discussed, as the first building block of the whole method that involves multiple branches in a single estimation technique, as explained in Section II.C.

The estimation of the network parameters relies on a line model for each branch, represented, in this paper, by the $\pi$-model of Fig. 1, where the shunt susceptances at the two ends of the line are neglected, as they are typically very small.

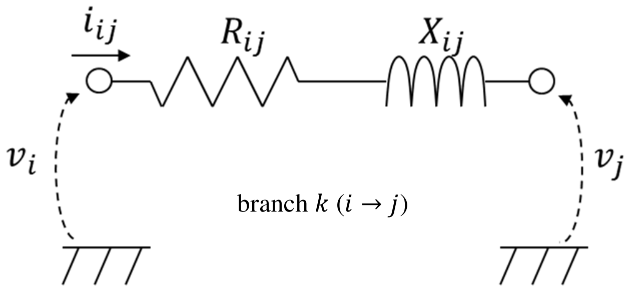


Fig. 1. Line model and line parameters for a network branch $k$.

The network is considered to be equipped with PMU measurement devices at each node and, in the following, with respect to the case depicted in Fig. 1, the starting and end node voltage phasors ($v_i$ and $v_j$) are measured, along with the current phasor on the line ($i_{ij}$), by two PMUs. All the measurements (indicated with the superscript $m$ in the following) are considered as independent and can be defined as:

$$\begin{aligned} v_i^m &= V_i^m e^{j\varphi_i^m} \\ v_j^m &= V_j^m e^{j\varphi_j^m} \\ i_{ij}^m &= I_{ij}^m e^{j\theta_{ij}^m} \end{aligned} \quad (1)$$

where both magnitude and phase angle measurements are affected by two contributions of the measurement errors, introduced by the measurement transducer and the PMU, respectively.

The errors are divided into their systematic and random parts, in order to emphasize the different short-term behavior of the two uncertainty sources. In the following, for the sake of a simpler notation, the transducer errors, defined by the accuracy class to which each installed transformer or divider belongs to, are considered to be the main source of systematic errors, while the PMU random errors are in the range given by instrument specifications.

With such an assumption, the measurement can be rewritten as a function of the reference values, as follows:

$$\begin{aligned} v_i^m &= V_i^m e^{j\varphi_i^m} = (1 + \xi_i + \xi_i^{PMU}) V_i e^{j(\varphi_i + \alpha_i + \alpha_i^{PMU})} \\ i_{ij}^m &= I_{ij}^m e^{j\theta_{ij}^m} = \left(1 + \eta_{ij} + \eta_{ij}^{PMU}\right) I_{ij} e^{j(\theta_{ij} + \psi_{ij} + \psi_{ij}^{PMU})} \end{aligned} \quad (2)$$

where $\xi_i$ and $\alpha_i$ are, respectively, the systematic relative magnitude and absolute phase angle measurement errors introduced by the voltage measurement chain (mainly the voltage transformer) at node $i$, while $\eta_{ij}$ and $\psi_{ij}$ are their counterpart for current measurements. The same symbols with superscript $PMU$ represent the random errors, that are mainly considered (without loss of generality) as introduced by the PMU channels, with analogous definitions. For the measurements of the end node $j$, a similar notation can be defined. In (2), the relative magnitude error contributions of both random and systematic origin are summed up, by neglecting the second order term given by the cascade of the two relative errors.

By writing the reference values in (2) as a function of the measurements and the errors, it is possible to express the voltage drop across the line as follows:

$$\begin{aligned} v_i - v_j &= V_i^m (1 - \xi_i - \xi_i^{PMU}) e^{j(\varphi_i^m - \alpha_i - \alpha_i^{PMU})} - \\ &\quad - V_j^m \left(1 - \xi_j - \xi_j^{PMU}\right) e^{j\left(\varphi_j^m - \alpha_j - \alpha_j^{PMU}\right)} \\ &= \left(R_{ij}^N \left(1 + \gamma_{ij}\right) + jX_{ij}^N \left(1 + \beta_{ij}\right)\right) \\ &\quad \left(1 - \eta_{ij} - \eta_{ij}^{PMU}\right) I_{ij}^m e^{j(\theta_{ij}^m - \psi_{ij} - \psi_{ij}^{PMU})} \end{aligned} \quad (3)$$

where $R_{ij}^N$ and $X_{ij}^N$ are the nominal resistance and reactance parameters and $\gamma$ and $\beta$ are the corresponding relative deviations from the nominal values. Equation (3) is obtained from (2) when all the relative magnitude errors of voltages and currents are small with respect to unity, as is usual.

Equation (3) is clearly nonlinear and imposes a constraint on the measurement errors given by the measurement values. When repeated PMU measurements on the same quantities are collected, several equations on the transducers errors are defined and it becomes important to separate the random contribution of the PMUs.

If $\xi_i$, $\xi_j$, $\alpha_i$, $\alpha_j$, $\eta_{ij}$, $\psi_{ij}$, $\gamma_{ij}$ and $\beta_{ij}$ are significantly smaller than one and the second order terms obtained by products of these variables can be neglected, it is possible to translate (3) in a linear equation (see the appendix for details) where all the random terms are summed up and, generally speaking, the measurement model becomes:

$$\mathbf{b}_m = \mathbf{A}_m \mathbf{x} + \boldsymbol{\epsilon}_m \quad (4)$$

where $\mathbf{b}_m$ and $\mathbf{A}_m$ are, respectively, the vector of the constant terms and the matrix given by measurement values, while $\mathbf{x}$ is the vector of the unknown terms and includes the random PMU deviations. In this context, the network parameter estimation, that is the estimation of the deviations from nominal values, must be obtained together with the transducer errors, that is by estimating $\mathbf{x} = \left[\xi_i, \alpha_i, \xi_j, \alpha_j, \eta_{ij}, \psi_{ij}, \gamma_{ij}, \beta_{ij}\right]^T$.

Even if multiple measurements of the phasors $v_i, v_j$ and $i_{ij}$ at different timestamps are exploited (corresponding to the rows of $\mathbf{A}_m$), the problem can appear underdetermined and it is thus useful to add the available prior information on the unknown parameters. It is thus possible to define additional constraints as follows:

$$\mathbf{0} = \mathbf{I}_8\mathbf{x} + \boldsymbol{\epsilon}_{\text{prior}} \tag{5}$$

where $\mathbf{0}$ is a 8x1 vector of zeros, $\mathbf{I}_8$ is the 8x8 identity matrix and $\boldsymbol{\epsilon}_{\text{prior}}$ is the vector representing the uncertainty of prior knowledge and is obtained from the maximum deviations associated to the unknowns (maximum relative magnitude and absolute phase angle errors of the transducers and maximum foreseen percent deviations of line resistance and reactance).

The estimated parameters $\hat{\mathbf{x}}$ can be obtained by means of weighted least square approach as:

$$\hat{\mathbf{x}} = \min_{\mathbf{x}} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_{\boldsymbol{\Sigma}_\epsilon} = \min_{\mathbf{x}} (\mathbf{b} - \mathbf{A}\mathbf{x})^T \boldsymbol{\Sigma}_\epsilon^{-1} (\mathbf{b} - \mathbf{A}\mathbf{x}) \tag{6}$$

where $\mathbf{b} = \begin{bmatrix}\mathbf{b}_m \\ \mathbf{0}\end{bmatrix}$ and $\mathbf{A} = \begin{bmatrix}\mathbf{A}_m \\ \mathbf{I}_8\end{bmatrix}$, and $\boldsymbol{\Sigma}_\epsilon$ is covariance matrix of the random vector $\boldsymbol{\epsilon} = \begin{bmatrix}\boldsymbol{\epsilon}_m \\ \boldsymbol{\epsilon}_{\text{prior}}\end{bmatrix}$. The solution is obtained analytically as:

$$\hat{\mathbf{x}} = (\mathbf{A}^T \boldsymbol{\Sigma}_\epsilon^{-1} \mathbf{A})^{-1} \mathbf{A}^T \boldsymbol{\Sigma}_\epsilon^{-1} \mathbf{b} \tag{7}$$

The WLS thus requires the definition of the matrix $\mathbf{A}$ and of the weighting matrix $\mathbf{W} = \boldsymbol{\Sigma}_\epsilon^{-1}$.

As mentioned above, $\mathbf{A}_m$ is the result of the linearization of (3). In particular, every set of measurements, at a given timestamp, defines two rows of $\mathbf{A}_m$, corresponding to the real and imaginary parts of the voltage drop constraints. Equation (8) (at the end of the page) shows the coefficients for $\mathbf{A}_m$ and $\mathbf{b}_m$, whose derivation is reported in the Appendix. The superscript "*r*" and "*x*" indicate the real and imaginary parts of the measured quantities, where "*m*" is dropped for simplicity.

Prior information is independent from the measurement errors and is directly defined by considering each parameter as a uniform random variable inside its variation range ($\sigma_{x_k} = \Delta x_k / 2\sqrt{3}$). For this reason, $\boldsymbol{\Sigma}_\epsilon = \begin{bmatrix}\boldsymbol{\Sigma}_{\epsilon_m} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{\epsilon_{\text{prior}}}\end{bmatrix}$ is block-diagonal. $\boldsymbol{\Sigma}_{\epsilon_{\text{prior}}} = diag([\sigma_{\xi_i}^2, \dots, \sigma_{\beta_{ij}}^2])$ is a diagonal matrix whose main diagonal includes the prior variances of the unknown parameters. $\boldsymbol{\Sigma}_{\epsilon_m}$ is described in detail in the following section devoted to measurement uncertainty definition.

It is important to note that $\boldsymbol{\Sigma}_{\epsilon_{\text{prior}}}$ reflects the prior, and limited, information available on the systematic errors in the knowledge of network parameters and transformers ratio and phase displacement.

### *B. Measurement Uncertainty and Weighting of the Residuals*

The covariance matrix $\boldsymbol{\Sigma}_{\epsilon_m}$ can be obtained by means of uncertainty propagation when all the uncertainty sources of PMU data are kept into account.

As detailed in the Appendix (see (A.6) and (A.7)), the random errors for each pair of equations can be expressed as:

$$\begin{bmatrix}\epsilon_{ij}^r \\ \epsilon_{ij}^x\end{bmatrix} = \mathbf{E}_{ij} \left[\xi_i^{PMU}, \xi_j^{PMU}, \alpha_i^{PMU}, \alpha_j^{PMU}, \eta_{ij}^{PMU}, \psi_{ij}^{PMU}\right]^T \tag{9}$$

It is thus clear that the 2x2 covariance matrix of $\boldsymbol{\epsilon}_{ij} = \begin{bmatrix}\epsilon_{ij}^r \\ \epsilon_{ij}^x\end{bmatrix}$ can be computed by classic GUM expression [8] as:

$$\boldsymbol{\Sigma}_{\epsilon_{ij}} = \mathbf{E}_{ij} \cdot \boldsymbol{\Sigma}^{\mathbf{PMU}} \cdot \mathbf{E}_{ij}^T \tag{10}$$

where $\boldsymbol{\Sigma}^{\mathbf{PMU}}$ represents the measurements covariance matrix that includes magnitude and phase angle deviations variances.

### *C. Simultaneous Estimation for Multiple Branches*

The concepts discussed in previous sections perfectly fit also to the case of multiple branch parameters estimations. A simple case with two adjacent branches is shown in Fig. 2.

In this case, the vector of parameters is doubled adding parameters for all the errors concerning three voltage phasor measurements, three current measurements (two branch-currents and the injected one) and four network parameters.

$$\mathbf{A}_m = \begin{bmatrix} & & & & \cdots & & & \\ V_i^r & -V_i^x & -V_j^r & V_j^x & \left(-R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x\right) & \left(R_{ij}^N I_{ij}^x + X_{ij}^N I_{ij}^r\right) & R_{ij}^N I_{ij}^r & -X_{ij}^N I_{ij}^x \\ V_i^x & V_i^r & -V_j^x & -V_j^r & \left(-R_{ij}^N I_{ij}^x - X_{ij}^N I_{ij}^r\right) & \left(-R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x\right) & R_{ij}^N I_{ij}^x & X_{ij}^N I_{ij}^r \\ & & & & \cdots & & & \end{bmatrix}$$

$$\mathbf{b}_m = \begin{bmatrix} \cdots \\ V_i^r - V_j^r - R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x \\ V_i^x - V_j^x - X_{ij}^N I_{ij}^r - R_{ij}^N I_{ij}^x \\ \cdots \end{bmatrix} \tag{8}$$

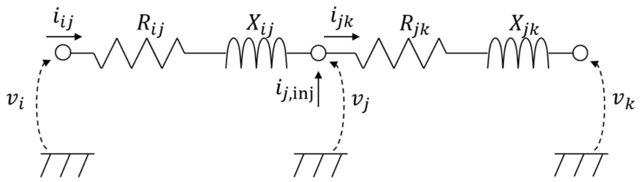


Fig. 2. Two branches schematic with current at the common point

The main difference is due to the current injection that gives an additional constraint due to Kirchoff's nodal rule at $k$. Equation (4) can be thus expanded adding new rows to $\mathbf{b}_m$ and $\mathbf{A}_m$, corresponding to the additional measurement equations. In particular, the following equation applies to node $j$ (with same notation as in Fig. 2):

$$i_{j,\text{inj}} = \sum_{k\in\Gamma_j} i_{jk} \tag{11}$$

where $\Gamma_j$ is the set of nodes connected to node $j$. After a few steps, reported in the Appendix, the constraints can be expressed as:

$$I^r_{j,\text{inj}} - \sum_{k\in\Gamma_j} I^r_{jk} = I^r_{j,\text{inj}}\eta_j - I^x_{j,\text{inj}}\psi_j - \sum_{k\in\Gamma_j} I^r_{jk}\,\eta_{jk} + \sum_{k\in\Gamma_j} I^x_{jk}\,\psi_{jk} + \epsilon^r_j \tag{12}$$

$$I^x_{j,\text{inj}} - \sum_{k\in\Gamma_j} I^x_{jk} = I^x_{j,\text{inj}}\eta_j + I^r_{j,\text{inj}}\psi_j - \sum_{k\in\Gamma_j} I^x_{jk}\,\eta_{jk} - \sum_{k\in\Gamma_j} I^r_{jk}\,\psi_{jk} + \epsilon^x_j \tag{13}$$

where the same notation as in (8) is adopted and $\eta_j$, $\psi_j$ are the error parameters of the measured current $i_{j,\text{inj}}$. From (12) and (13) it is clear that the additional constraints involve all the current measurements of the incoming/outgoing branches.

When a portion of a network is considered, it is very common to have also zero-injection nodes, where the current balance is zero. This case can be treated as the previous one, but there are no additional unknown parameters and (12), (13) have the same shape but with the first term of left hand side of the equality equal to zero.

Both in the case of injected or zero injection currents, the covariance matrix of the random errors $\epsilon^r_j$ and $\epsilon^x_j$ using an equation similar to (10), where a new matrix $\mathbf{E}_j$ , which represents the Jacobian of the polar to rectangular transformation, is considered. It is important to highlight that, as clear from (A.10) and (A.11), $\mathbf{E}_j$ links different unknown parameters in $\mathbf{x}$, and thus in the overall computation of $\mathbf{W} = \mathbf{\Sigma}_{\boldsymbol{\epsilon}}^{-1}$ many correlations arise.

## III. Tests and Results

The tests have been performed in simulation environment, considering a PMU-based monitoring architecture with the aforementioned characteristics, that is phasor measurements of node voltages and of branch and node currents. The magnitude and phase angle random errors for both voltages and currents have been chosen according to typical PMU accuracies under steady-state conditions: a maximum TVE error of 0.1 % is used and translated into 0.1 % maximum amplitude error and $0.1 \cdot 10^{-2}$ rad (0.1 crad) for maximum phase angle error.

The systematic errors are considered to be the effect of voltage and current transformers (VTs and CTs). VTs and CTs belong to 0.5-class and thus have the following limits for the errors (see parts 2 and 3 of the standard IEC 61869 [9], [10]):

- VT: 0.5 % for ratio error and 0.6 crad for phase displacement
- CT: 0.5 % for ratio error and 0.9 crad (considering a current that is greater than 50 % of the rated value) for phase displacement

Such values are assumed to be the maximum absolute values of the parameters $\xi, \alpha, \eta, \psi$. Due to the lack of a-priori information, the parameters are assumed to be uniformly distributed and a variance equal to one-third of the squared maximum deviation is adopted for the WLS.

The considered network is a portion of the 95-node network (used in [11]) composed only by the first 9 nodes. The network parameters are considered to vary in the range $\pm 30\%$ around the nominal values, as often assumed by operators [4]. In this respect, a complete analysis of the limits of the first order approximation that leads from (3) to (4) is beyond the scope of this paper. Actually, when needed, iterative solutions could be also applied.

The test is performed by considering 10 different operating conditions of the network (the active and reactive power of the loads can vary randomly with a maximum deviation of 30%). For each operating condition, 10 measurement sets are obtained from the PMUs, corresponding to 10 different timestamps. When different conditions are considered, the problem in (4) is better determined, but the underlying assumption is that both systematic errors in VT and CT and in line parameter deviations can be considered constant. It is particularly important to underline that the assumption holds true only when the variation range of currents is not too wide (it can be checked in real-time from measured values).

Fig. 3 and Fig. 4 show the results of the estimations obtained for resistance and reactance parameters for all the branches of the sample network. The root mean of the average square relative errors is reported for every branch when the estimation is performed for one branch at a time (as described in Section II.A) or simultaneously on the whole network (see Section II.C). The statistics are obtained by performing 1000 Monte Carlo trials, extracting the line parameters from their prior (the corresponding statistics are represented by the yellow dash-dotted line in the figures). In each trial, the systematic measurement errors are added to the reference values obtained from a powerflow computation.

From the reported results, it is clear how the impedance estimation for specific branches benefits from the presence of additional constraints available in simultaneous estimation. In particular, as a first outcome, it can be observed that the estimation for a single branch improves the prior knowledge by including multiple network conditions and multiple measurements in the estimation process (as testified by the differences between the red crosses and the yellow dash-dotted line in Figs. 3 and 4). Furthermore, simultaneous estimation of multiple branches allows strongly refining the parameters knowledge.

Other branches correspond to leaves (2, 5 and 7) or short feeders and mainly rely on local measurement for the estimation, making the uncertainty reduction almost negligible.

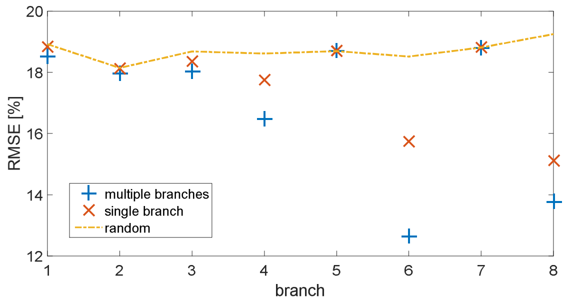


Fig. 3. Root Mean Square Error for branch resistance estimations with single and multiple branch algorithms

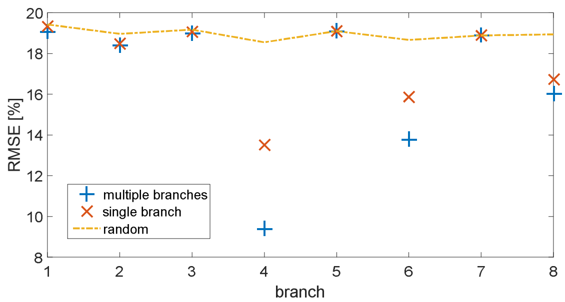


Fig. 4. Root Mean Square Error for branch reactance estimations with single and multiple branch algorithms

## IV. Conclusions

In this paper, an approach that includes transducer errors and network line parameters in a single PMU-based estimation method, which properly considers different uncertainty sources, has been presented. The main outcome is the possibility to improve the estimation by using multiple measurements collected under different network conditions and by simultaneously calculating multiple branch impedances, thus correcting the large deviations of prior distributions, thanks also to the network topology and the available distributed measurement system.

## Appendix

### Derivation of Measurement and Weighting Matrix

When small deviations are considered for measurement errors and for network parameters it is possible to substitute the exponentials with their first-order truncated McLaurin series and (3) results as follows:

$$\begin{aligned} &V_i^m(1-\xi_i-\xi_i^{PMU})e^{j\varphi_i^m}\left(1-j(\alpha_i+\alpha_i^{PMU})\right) \\ &-V_j^m(1-\xi_j-\xi_j^{PMU})e^{j\varphi_j^m}\left(1-j(\alpha_j+\alpha_j^{PMU})\right) \\ &\qquad=\left(R_{ij}^N(1+\gamma_{ij})+jX_{ij}^N(1+\beta_{ij})\right) \\ &\quad(1-\eta_{ij}-\eta_{ij}^{PMU})I_{ij}^m e^{j\theta_{ij}^m}\left(1-j(\psi_{ij}+\psi_{ij}^{PMU})\right) \end{aligned} \tag{A.1}$$

and, using Eulero's formulas, neglecting second order terms and splitting real and imaginary parts of the complex-valued expression, after a few passages:

$$\begin{aligned} &V_i^m\cos\varphi_i^m-V_j^m\cos\varphi_j^m \\ &-R_{ij}^N I_{ij}^m\cos\theta_{ij}^m+X_{ij}^N I_{ij}^m\sin\theta_{ij}^m \\ &\qquad=V_i^m\cos\varphi_i^m\,\xi_i-V_j^m\cos\varphi_j^m\,\xi_j \\ &\qquad-V_i^m\sin\varphi_i^m\,\alpha_i+V_j^m\sin\varphi_j^m\,\alpha_j \\ &\qquad+\left(-R_{ij}^N I_{ij}^m\cos\theta_{ij}^m+X_{ij}^N I_{ij}^m\sin\theta_{ij}^m\right)\eta_{ij} \\ &\qquad+\left(R_{ij}^N I_{ij}^m\sin\theta_{ij}^m+X_{ij}^N I_{ij}^m\cos\theta_{ij}^m\right)\psi_{ij} \\ &\qquad+R_{ij}^N I_{ij}^m\cos\theta_{ij}^m\,\gamma_{ij}-X_{ij}^N I_{ij}^m\sin\theta_{ij}^m\,\beta_{ij}+\epsilon_{ij}^r \end{aligned} \tag{A.2}$$

$$\begin{aligned} &V_i^m\sin\varphi_i^m-V_j^m\sin\varphi_j^m \\ &-X_{ij}^N I_{ij}^m\cos\theta_{ij}^m-R_{ij}^N I_{ij}^m\sin\theta_{ij}^m \\ &\qquad=V_i^m\sin\varphi_i^m\,\xi_i-V_j^m\sin\varphi_j^m\,\xi_j \\ &\qquad+V_i^m\cos\varphi_i^m\,\alpha_i-V_j^m\cos\varphi_j^m\,\alpha_j \\ &\qquad+\left(-R_{ij}^N I_{ij}^m\sin\theta_{ij}^m-X_{ij}^N I_{ij}^m\cos\theta_{ij}^m\right)\eta_{ij} \\ &\qquad+\left(-R_{ij}^N I_{ij}^m\cos\theta_{ij}^m+X_{ij}^N I_{ij}^m\sin\theta_{ij}^m\right)\psi_{ij} \\ &\qquad+R_{ij}^N I_{ij}^m\sin\theta_{ij}^m\,\gamma_{ij}+X_{ij}^N I_{ij}^m\cos\theta_{ij}^m\,\beta_{ij}+\epsilon_{ij}^x \end{aligned} \tag{A.3}$$

where $\epsilon^r$ and $\epsilon^x$ are the random errors due to all the PMU measurement errors.

By expressing measured phasors in rectangular coordinates, the following expressions are obtained (the $m$ superscript is dropped for the sake of clearness):

$$\begin{aligned} &V_i^r-V_j^r-R_{ij}^N I_{ij}^r+X_{ij}^N I_{ij}^x \\ &\qquad=V_i^r\xi_i-V_j^r\xi_j-V_i^x\alpha_i+V_j^x\alpha_j \\ &\qquad+\left(-R_{ij}^N I_{ij}^r+X_{ij}^N I_{ij}^x\right)\eta_{ij}+\left(R_{ij}^N I_{ij}^x+X_{ij}^N I_{ij}^r\right)\psi_{ij} \\ &\qquad+R_{ij}^N I_{ij}^r\gamma_{ij}-X_{ij}^N I_{ij}^x\beta_{ij}+\epsilon_{ij}^r \end{aligned} \tag{A.4}$$

$$V_i^x - V_j^x - X_{ij}^N I_{ij}^r - R_{ij}^N I_{ij}^x$$
$$= V_i^x \xi_i - V_j^x \xi_j + V_i^r \alpha_i - V_j^r \alpha_j$$
$$+\left(-R_{ij}^N I_{ij}^x - X_{ij}^N I_{ij}^r\right)\eta_{ij} + \left(-R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x\right)\psi_{ij}$$
$$+R_{ij}^N I_{ij}^x \gamma_{ij} + X_{ij}^N I_{ij}^r \beta_{ij} + \epsilon_{ij}^x \tag{A.5}$$

where the superscripts "$r$" and "$x$" indicate real and imaginary parts. From (A.4) and (A.5) (8) is derived. With the same notation, the random errors can be represented as:

$$\epsilon_{ij}^r = V_i^r \xi_i^{PMU} - V_j^r \xi_j^{PMU} - V_i^x \alpha_i^{PMU} + V_j^x \alpha_j^{PMU} + \left(-R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x\right)\eta_{ij}^{PMU} + \left(R_{ij}^N I_{ij}^x + X_{ij}^N I_{ij}^r\right)\psi_{ij}^{PMU} \tag{A.6}$$

$$\epsilon_{ij}^x = V_i^x \xi_i^{PMU} - V_j^x \xi_j^{PMU} + V_i^r \alpha_i^{PMU} - V_j^r \alpha_j^{PMU} + \left(-R_{ij}^N I_{ij}^x - X_{ij}^N I_{ij}^r\right)\eta_{ij}^{PMU} + \left(-R_{ij}^N I_{ij}^r + X_{ij}^N I_{ij}^x\right)\psi_{ij}^{PMU} \tag{A.7}$$

Equations (A.6) and (A.7) allow to express the random errors as a linear function of the random PMU errors and to define $\mathbf{E}_{ij}$ in (9). As a consequence, $\mathbf{\Sigma}_{\epsilon_{ij}}$ can be computed.

Concerning current injection measurements, starting from (11) and considering small deviations as above, the following relationships hold:

$$I_{j,\mathrm{inj}} \cos\theta_j - \sum_{k\in\Gamma_j} I_{jk} \cos\theta_{jk}$$
$$= I_{j,\mathrm{inj}} \cos\theta_j \,\eta_j - I_{j,\mathrm{inj}} \sin\theta_j \,\psi_j$$
$$- \sum_{k\in\Gamma_j} I_{jk} \cos\theta_{jk} \,\eta_{jk} + \sum_{k\in\Gamma_j} I_{jk} \sin\theta_{jk} \,\psi_{jk} + \epsilon_j^r \tag{A.8}$$

$$I_{j,\mathrm{inj}} \sin\theta_j - \sum_{k\in\Gamma_j} I_{jk} \sin\theta_{jk}$$
$$= I_{j,\mathrm{inj}} \sin\theta_j \,\eta_j + I_{j,\mathrm{inj}} \cos\theta_j \,\psi_j$$
$$- \sum_{k\in\Gamma_j} I_{jk} \sin\theta_{jk} \,\eta_{jk} - \sum_{k\in\Gamma_j} I_{jk} \cos\theta_{jk} \,\psi_{jk} + \epsilon_j^x \tag{A.9}$$

where:

$$\epsilon_j^r = I_{j,\mathrm{inj}} \cos\theta_j \,\eta_j^{PMU} - I_{j,\mathrm{inj}} \sin\theta_j \,\psi_j^{PMU} - \sum_{k\in\Gamma_j} I_{jk} \cos\theta_{jk} \,\eta_{jk}^{PMU} + \sum_{k\in\Gamma_j} I_{jk} \sin\theta_{jk} \,\psi_{jk}^{PMU} \tag{A.10}$$

$$\epsilon_j^x = I_{j,\mathrm{inj}} \sin\theta_j \,\eta_j^{PMU} + I_{j,\mathrm{inj}} \cos\theta_j \,\psi_j^{PMU} - \sum_{k\in\Gamma_j} I_{jk} \sin\theta_{jk} \,\eta_{jk}^{PMU} - \sum_{k\in\Gamma_j} I_{jk} \cos\theta_{jk} \,\psi_{jk}^{PMU} \tag{A.11}$$

From (A.8)-(A.11), (12) and (13) can be easily derived and $\mathbf{E}_j$ can be obtained, thus allowing to set the weights to be used in the WLS procedure.

## REFERENCES


[1] A. Monti, C. Muscas, F. Ponci, et al. "Phasor Measurement Units and Wide Area Monitoring Systems", 1st ed., A. Monti, C. Muscas, and F. Ponci, Eds. Elsevier, Academic Press, 2016.

[2] X. Tangsheng, W. Li, Z. Song, Z. Linlin, K. Jin, and C. Wei, "Study on online calculation method of transmission line parameters," in 11th IET International Conference on Developments in Power Systems Protection (DPSP 2012), 2012, pp. 1–4.

[3] R. Matica, V. Kirincic, S. Skok, and A. Marusic, "Transmission line impedance estimation based on PMU measurements," in Eurocon 2013, 2013, pp. 1438–1444.

[4] G. L. Kusic and D. L. Garrison, "Measurement of transmission line parameters from SCADA data," in IEEE PES Power Systems Conference and Exposition, 2004., 2004, pp. 440–445 vol.1.

[5] C. Muscas, M. Pau, P. A. Pegoraro and S. Sulis, "Uncertainty of Voltage Profile in PMU-Based Distribution System State Estimation," in IEEE Transactions on Instrumentation and Measurement, vol. 65, no. 5, pp. 988-998, May 2016.

[6] D. Shi, D. J. Tylavsky, and N. Logic, "An Adaptive Method for Detection and Correction of Errors in PMU Measurements," IEEE Trans. Smart Grid, vol. 3, no. 4, pp. 1575–1583, Dec. 2012.

[7] C. M. Roberts, C. M. Shand, K. W. Brady, E. M. Stewart, A. W. McMorran, and G. A. Taylor, "Improving distribution network model accuracy using impedance estimation from micro-synchrophasor data," in 2016 IEEE Power and Energy Society General Meeting (PESGM), 2016, pp. 1–5.

[8] ISO/IEC 98-3:2008 "Uncertainty of measurement - Part 3: Guide to the expression of uncertainty in measurement", 2008 (reissue of the 1995 version of the GUM).

[9] IEC "EN 61869-2:2012-11: Instrument transformers - Part 2: Additional requirements for current transformers", 2012.

[10] IEC, "IEC 61869-3:2011-10: Instrument transformers Part 3: Additional requirements for inductive voltage transformers, 2011.

[11] Marco Pau "State Estimation in Electrical Distribution Systems", Ph.D. dissertation, Department of Electrical and Electronic Engineering, University of Cagliari, Cagliari, Italy, 2015.